Question 1

1. Prove: $n = 1 - \frac{\gamma_d}{G_s \gamma_w}$
   Given: $n = \frac{V_v}{V_t}, \gamma_d = \frac{W_s}{V_t}, G_s = \frac{\gamma_s}{\gamma_w}, V_t = V_v + V_s$
2. Prove: $\gamma_d = \gamma_{sat} - \frac{e}{1+e}\gamma_w$
   Given: $e = \frac{V_v}{V_s}, \gamma_d = \frac{W_s}{V_t}, \gamma_{sat} = \frac{W_s + W_w}{V_t}$

Solutions

Part 1

We start with the definition of porosity and then move on a few simple steps:

$$n = \frac{V_v}{V_t} = \frac{V_t - V_s}{V_t} = 1 - \frac{V_s}{V_t}$$

We then multiply both the nominator and denominator by $1/W_s$ which is essentially multiplying by one, hence still mathematically acceptable, and get:

$$n = 1 - \frac{V_s}{V_t} = 1 - \frac{\frac{W_s}{V_t}}{\frac{W_s}{V_s}}$$

Since we know that $\frac{W_s}{V_s} = \gamma_s$ and similarly we know that $\gamma_s = G_s\gamma_w$ we can write:

$$n = 1 - \frac{\frac{W_s}{V_t}}{G_s \gamma_w}$$

And finally, knowing that $\frac{W_s}{V_t} = \gamma_d$ we can find our desired result:

$$n = 1 - \frac{\gamma_d}{G_s \gamma_w}$$

Part 2

This problem was more complex then the first in that it requiered a very tricky assumption that may not be entirely obvious at first.

We start, similarly to Part 1, with the actual defition of our desired value, here γ_d :

$$\begin{aligned} \gamma_d &= \frac{W_s}{V_t} = \frac{W_t - W_w}{V_t} = \frac{W_t}{V_t} - \frac{V_v \gamma_w}{V_t} \\ &= \gamma_{sat} - \frac{\frac{V_v}{V_s}}{\frac{V_s + V_v}{V_s}} \end{aligned}$$

Here, in the last two steps, we take the assumption that we are working on a saturated soil. In that case in order to be consistant we have to assume all the volume of voids are also volume of water, $V_v = V_w$, and that the total weight is actually the total saturated weight, $\gamma = \gamma_{sat}$. We wish to use volume of voids instead of volume of water as the volume of void is connected to the void ratio in which we are interested.

Once this step is clearly understood, we can simply follow through to the end result:

$$\begin{aligned}
\gamma_d &= \gamma_{sat} - \frac{\frac{V_v}{V_s}}{1 + \frac{V_v}{V_s}}\gamma_w \\
&= \gamma_{sat} - \frac{e}{1+e}\gamma_w
\end{aligned}$$

Question 2

In a natural state, a moist soil has a volume of $98.2cm^3$ and has a mass of 181.5g. The oven dry mass of the soil is 170g. If G_s is 2.67, using a phase diagram, calculate:

1. Moisture content (%)
2. Wet unit weight (kN/m^3)
3. Dry unit weight (kN/m^3)
4. Void ratio
5. Porosity
6. Degree of saturation

Solution

Moisture Content

We can get the moisture content easily from its definition as follows:

$$\omega = \frac{M_w}{M_s} = \frac{M_t - M_s}{M_s} = \frac{181.5g - 170g}{170g} * 100\% = 6.76\%$$

Wet Unit Weight

The wet Unit Weight is again easily obtained by the definition:

$$\gamma = \frac{W}{V} = \frac{181.5g * 9.806m/s^2}{98.2cm^3} = 18.12kN/m^3$$

Dry Unit Weight

Again, this part is very easily obtained as follows:

$$\gamma_d = \frac{W_s}{V} = \frac{M_s * g}{V} = \frac{170g * 9.806m/s^2}{98.2cm^3} = 16.9kN/m^3$$

0.0.1 Void Ratio

Here, we have to play a bit with algebra to get something we can use, we can proceed as follows:

$$\begin{aligned} e &= \frac{V_v}{V_s} = \frac{V_t - V_s}{\frac{M_s}{G_s \rho_w}} \\ &= \frac{V_t - \frac{M_s}{G_s \rho_w}}{\frac{M_s}{G_s} \rho_w} \\ &= \frac{98.2cm^3 - \frac{170g}{2.65 * 1g/cm^3}}{\frac{170g}{2.65 * 1g/cm^3}} \\ &= 0.5307 \end{aligned}$$

For future reference, we can also pre-compute the volume of voids as

$$V_v = \frac{M_s}{G_s \rho_w} = 34cm^3$$

Porosity

Now that we have void ratio (and hence volume of voids) we can easily find porosity as:

$$n = \frac{V_v}{V} = \frac{34cm^4}{98.2cm^3} = 0.346$$

0.0.2 Degree of Saturation

Finally, we can get degree of saturation knowing the volume of water (from the difference in dry and wet weights) and the volume of voids.

$$S_r = \frac{V_w}{V_v} = \frac{M_t - M_s}{\rho_w V_v} = \frac{181.5g - 170g}{1g/cm^3 * 34cm^3} = 0.338$$

Question 3

A soil has a unit weight of 19.6 kN/m^3 and a water content of 10%. The value of G_s is 2.65. Calculate the dry density, void ratio, degree of saturation, and air content. What is the density and water content if the soil is fully saturated?

Solution

Dry Density

Here we can use a convenient formula:

$$\gamma_d = \frac{\gamma}{1+\omega} = 17.81 kN/m^3$$

Void Ratio

We can also use another convenient trick for this question going as follows:

$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

Solving for e, we can thus get:

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = 0.352$$

Degree of Saturation

This can be done in a fairly standard way. Do keep in mind that all these formulas used are either in your formula sheet or easily derived using the methods covered in the past 2 questions.

$$S_r = \frac{\omega G_s}{e} = 0.7528$$

Air Content

For the air content, we proceed as follows:

$$A = \frac{e - \omega G_s}{1+e} = 0.079$$

What happens when fully saturated

When fully saturated, we know the degree of saturation will be unity so we can reuse the relation from before and find omega as shown next where $S_r = \omega G_s/e$ becomes:

$$\omega = e/G_s = 0.1328$$

As for the density, well we can reuse our dry density formula and again obtain something useful:

$$\gamma_d = \frac{\gamma_{sat}}{1+\omega_{sat}}$$

or, rearanging,

$$\gamma_{sat} = \gamma_d\left(1+\omega_{sat}\right) = 20.175kN/m^3$$

Question 4

The in-situ void ratio of a soil is e=0.59 and G_s=2.61. Calculate the porosity, dry unit weight, and the saturated unit weight. What would the moist unit weight be if the degree of saturation was S=0.55?

Solution

Porosity

This question differs from others in that we start with a void ratio. As such it is a tad easier to get parameters, like in this case porosity:

$$n = \frac{e}{1+e} = 0.371$$

Dry Unit Weight

We revisit the previously used formula in Question 3

$$\gamma_d = \frac{G_s}{1+e}\gamma_w = 16.35kN/m^3$$

Saturated Unit Weight

We introduce here a new formula for computing the saturated unit weight. As all previous formulas, it is easily derived using the tricks we learned before.

$$\gamma_{sat} = \frac{G_s + e}{1 + e}\gamma_w = 19.99kN/m^3$$

Properties at 55% Saturation

To get the moisture content, we can simply rearrange the degree of saturation formula we have been using to get :

$$\omega = \frac{S_r e}{G_s} = 0.156$$

And we can get the unit density using the following method, which is a derivation of our previously used methods:

$$\gamma = \frac{G_s\,(1 + \omega)}{1 + e}\gamma_w = 19.20kN/m^3$$

Question 5

What is the void ratio for the face centered cubic packing (FCC) given the size of a side on the unit cell is $a = 2R\sqrt{2}$? What is the coordination number of this packing?

Solution

Please refer to the structure of an FCC unit cell for this derivation to make sense:

$$\begin{aligned}
V_t &= (2R\sqrt{(2)})^3 = 16 * 2^{1/2} * R^3 \\
V_s &= 4 * V_{sphere} = 4 * (4/3)\pi R^3 \\
V_v &= V_t - V_s = (3 * 2^{1/2} - \pi) * (16/3) * R^3 \\
e &= \frac{V_v}{V_s} = 0.35
\end{aligned}$$